Physics of Atmospheres and Oceans: Class Question Sheets

COMPARATIVE PLANETARY ATMOSPHERES

PLA.1 Formation of planets

What are the key properties of the solar system to be explained by any theory of its origin? What are the main features of the two main classes of theory, and the main reasons for discounting the less popular of these?

Show that the energy liberated during the collapse of a sphere of mass M and uniform density from infinity to radius R is given approximately by:

$$E = -\frac{GM^2}{R}$$

Using this result, derive the expression

$$M_J \simeq \left(\frac{kT}{G\mu m_H}\right)^{\frac{3}{2}} \frac{1}{\rho^{\frac{1}{2}}}$$

for the critical mass for gravitational collapse of an isothermal cloud of mass M_J, temperature T, density ρ (k = Boltzmann's constant, μ = molecular weight, m_H = mass of hydrogen atom).

If T=20 K, ρ=10^{10} H atoms m^{-3}, evaluate M_J and the time scale for collapse and comment on the results.

[14]

Assuming Jupiter formed at the same time as the Sun, the total gravitational energy released by its formation is given roughly by the equation derived above

$$E = -\frac{GM^2}{R}$$

where M is the mass of Jupiter and R its radius. Assuming this is all converted to thermal energy, give a crude estimate for the internal temperature of Jupiter by equating this energy to the mean internal energy $U = nC_vT$

(The mass of Jupiter is 1.9×10^{27} kg, the mean molecular weight 2.27 g, the heat capacity at constant volume $C_v = 2.5R$ J mol^{-1} K^{-1}, and the radius is approximately 70 000 km).

Jupiter is observed to be still releasing its energy of formation at a rate of approximately 5.4 W m^{-2}. Given that the mean radius of the planet is 70 000 km, calculate the rate at which the radius of Jupiter shrinks (express your answer in terms of mm/year).

Discuss the limitations of this rather approximate calculation.

[6]
Total: [20]

PLA.2 Atmospheric Evolution. Escape Processes

Assuming an isothermal atmosphere, show that the number of molecules per unit area above a level of altitude z_0 is:

$$N(z_0) = \int_{z_0}^{\infty} n(z)\, dz = n(z_0)H$$

where $n(z)$ is the number density (molecules/m^3) of the atmosphere at altitude z, H is the scale height given by $H = RT/Mg$, R is the gas constant, T is the mean temperature, M is the mean molecular weight, and g is the mean gravitational acceleration.

Define what is meant by the *exobase* or *critical level* z_c for thermal escape.

[5]

Assuming a Maxwell-Boltzmann distribution, the probability that a molecule will have a speed in the range c to $c + dc$ is:

$$P(c)\, dc = 4\left(\frac{\alpha^3}{\pi}\right)^{\frac{1}{2}} c^2 \exp\left(-\alpha c^2\right)\, dc$$

where $\alpha = m/2kT$. The upward flux of molecules at the exobase with speeds in the range c to $c+dc$ is then:

$$dF = \frac{1}{4} n(z_c) c P(c)\, dc$$

Assuming that all such molecules with speed greater than the escape velocity v_e at the exobase will escape the atmosphere, show that the escaping Jeans flux is:

$$F_{\text{Jea}} = \frac{1}{2} n(z_c) \left(\frac{\alpha}{\pi}\right)^{\frac{1}{2}} \left(v_e^2 + \frac{1}{\alpha}\right) \exp\left(-\alpha v_e^2\right)$$

or

$$F_{\text{Jea}} = \frac{n(z_c) U}{2\sqrt{\pi}} \left(1 + \frac{v_e^2}{U^2}\right) \exp\left(-v_e^2/U^2\right)$$

where the most probable speed $U = \sqrt{2kT/m}$.

[5]

By considering the rate of loss of molecules from a cylinder of unit cross-sectional area above the exobase, show that:

$$F_{\text{Jea}} = -\frac{dN(z_c)}{dt} = \frac{\beta}{H_c} N(z_c)$$

where $\beta = U/(2\sqrt{\pi})e^{-\lambda}(\lambda+1)$, $\lambda = v_e/U$, H is the scale height and $N(z_c)$ is the total number of molecules per unit area above the exobase. Hence, show that that the characteristic escape time $\tau_\varepsilon = H_c/\beta$ (Alternatively we can define an expansion velocity $v_{\text{ex}} = 1/\beta$).

[5]

Estimate the characteristic escape time for atomic hydrogen and oxygen on Mars and Jupiter.

	Thermospheric Temperature [K]	Radius at Exobase [km]	Gravitational Accn. at Exobase [m s^{-2}]
Mars	365	3 590	3.32
Jupiter	155	69 500	26.2

[5]
Total: [20]

PLA.3 Radiative Balance and Thermal Structure

Given the information in the table below, estimate the following:

Radius of the Sun (Mkm)	0.7			
Effective temperature of the Sun (K)	6000			
	Venus	Earth	Mars	Jupiter
Distance from Sun (Mkm)	108	150	228	780
Bolometric Albedo, A	0.76	0.4	0.15	0.27
Atmospheric Mean Molecular weight	44	29	44	2.27
Acceleration due to gravity (m s^{-2})	8.88	9.81	3.73	26.2
Specific heat (J kg^{-1} K^{-1})	830	1005	830	12800
Surface temperature (K)	730	290	235	N/A

(a) the power output of the Sun in Watts

[2]

(b) the stratospheric temperature on Venus

[5]

(c) the mean height of the tropopause above the surface of Mars.

[8]

Calculate the dry adiabatic lapse rates in the atmospheres of Venus, Earth, Mars and Jupiter. Calculate also the stratospheric temperatures. How do your estimates compare with the values observed?

[5]
Total: [20]

PLA.4 Cloud Formation

The Clausius-Clapeyron equation for the pressure-temperature relationship along a liquid-vapour or gas-vapour boundary is:

$$\frac{dp}{dT} = \frac{Lp}{RT^2}$$

where L is the molar heat of vapourisation (J mol^{-1}), and R is the universal gas constant. If at some point on the phase boundary, $p = p_1$ and $T = T_1$, and assuming that L does not vary with temperature, show that:

$$\ln\left(\frac{p}{p_1}\right) = -\frac{L}{R}\left(\frac{1}{T} - \frac{1}{T_1}\right)$$

[2]

In the region of Jupiter's atmosphere where water and ammonia clouds are predicted to form, the temperature structure is well represented by a dry adiabatic lapse rate. Defining a reference pressure level p_0 at temperature T_0 and measuring all heights with respect to this level, the temperature may be written as $T = T_0 - \Gamma z$ where Γ is the lapse rate. Derive the hydrostatic equation for such an atmosphere and show that:

$$\frac{p}{p_0} = \left(\frac{T}{T_0}\right)^{Mg/\Gamma R} = \left(\frac{T}{T_0}\right)^{Mc_p/R} = \left(\frac{T}{T_0}\right)^{C_p/R}$$

where M is the molecular weight of the atmosphere, c_p is the heat capacity per unit mass at constant pressure, and C_p is the heat capacity per mole at constant pressure.

[5]

Consider a parcel of air rising from the deep atmosphere of Jupiter containing a volume mixing ratio x of water vapour. The partial pressure of water at any point will then be:

$$p_w = xp = xp_0\left(\frac{T}{T_0}\right)^{C_p/R}$$

This water will start to condense where its partial pressure exceeds the saturated vapour pressure. Hence the condensation level occurs where:

$$p_w = xp_0\left(\frac{T}{T_0}\right)^{C_p/R} = p_1 \exp\left(-\frac{L}{R}\left(\frac{1}{T} - \frac{1}{T_1}\right)\right)$$

Given that for water vapour, $L = 2.5 \times 10^6$ J kg^{-1}, and that the saturated vapour pressure of water vapour at 273 K is 610 Pa, take logs of both sides of this equation and solve for T either graphically or by another method. Calculate the corresponding atmospheric pressure and height (referring to pressure level p_0).

(In the Jovian atmosphere the temperature at the pressure level 10^5 Pa is 165 K, the volume mixing ratio of water vapour in the deep atmosphere is approximately 10^{-3}, $g = 26.2$ m s^{-2}, $C_p = 3.5R$ and the molecular weight is 0.00227 kg).

[13]

Total: [20]