Physics of Atmospheres and Oceans: Class Question Sheets

DYNAMICS of PLANETARY ATMOSPHERES

DPA.1

The meridional component of the equation of motion on a spherical planet of radius a and angular velocity Ω is

$$\frac{Dv}{Dt} + \frac{u^2 \tan\phi}{a} + \frac{wv}{a} + 2\Omega u \sin\phi = -\frac{1}{\rho a}\frac{\partial p}{\partial \phi}$$

where ϕ is latitude, p is pressure, ρ is the density and (u, v, w) are the eastward, northward and vertical components of velocity, respectively. What balance of terms in the above equation represents *cyclostrophic balance*?

Titan, the principal satellite of Saturn, rotates very slowly about its axis (period $\approx$ 16 days) such that the Coriolis acceleration can be neglected. Assuming hydrostatic balance, and neglecting vertical variations of temperature, show that the thermal wind equation on Titan is approximately

$$2u\frac{\partial u}{\partial z} = -\frac{g}{T\tan\phi}\frac{\partial T}{\partial \phi} \ .$$

Measurements in Titan's atmosphere indicate a temperature difference between equator and high latitudes of 10 K, independent of height up to 60 km altitude. Estimate the maximum zonal wind speed, given that the atmosphere is at a mean temperature of 90 K and that $g = 1.35\ \mathrm{m\,s^{-2}}$.

DPA.2

The steady state equations for an incompressible fluid flowing over topography are

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} - fv = -\frac{1}{\rho_0}\frac{\partial p}{\partial x} \ ,$$

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + fu = -\frac{1}{\rho_0}\frac{\partial p}{\partial y} \ ,$$

$$\frac{\partial}{\partial x}(hu) + \frac{\partial}{\partial y}(hv) = 0 \ ,$$

where h is the depth of the fluid, u, v are the zonal and meridional velocities, f the Coriolis parameter, ρ_0 the density (taken to be constant) and p the pressure. Taking the curl of the first two equations, show that they imply the conservation of potential vorticity q following the horizontal motion, where

$$q = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} + f\right)\Big/ h \ .$$

Introduce a suitable streamfunction ψ, and show that q-conservation may be written as

$$J(\psi, q) \equiv \frac{\partial \psi}{\partial x}\frac{\partial q}{\partial y} - \frac{\partial \psi}{\partial y}\frac{\partial q}{\partial x} = 0 \ .$$

Hence show that weak flow will occur parallel to contours of f/h. Find the form of the weak flow which would occur on a smooth planet ($h = h_0$), for which $\psi = Kf$, where K and h_0 are constants, and $f = 2\Omega\sin\phi$, where ϕ is the latitude.

DPA.3

Take the nonlinear free-surface equations for non-divergent flow $Du - fv + gh_x = 0$, $Dv + fu + gh_y = 0$, $u_x + v_y = 0$, where

$$D = \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y}$$

is the horizontal advective derivative. Retain the β-effect. Show that $D(\xi + f) = 0$; what is the physical interpretation of this equation?

Now consider small non-divergent disturbances to a steady, uniform zonal flow $(U, 0)$. Put $u = U + u'$, $v = v'$; introduce a disturbance stream function ψ' and show that

$$\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)(\psi'_{xx} + \psi'_{yy}) + \beta\psi'_x = 0$$

when quadratic terms in ψ' are neglected. Hence derive the dispersion relation for non-divergent Rossby waves in the presence of the basic zonal flow. Under what conditions is the phase speed (ω/k) eastwards?

Suppose that the zonal wavenumber k is specified, and that l is real. Show that stationary waves (with $\omega = 0$) can only exist if U is eastwards and less than βk^{-2} in magnitude.

The surface topography in the northern hemisphere of Mars consists of a pair of ridges oriented north-south and 180° apart in longitude. One corresponds to the Tharsis Plateau centred at longitude 100° W, the other to Syrtis at about 80° E. Consider a latitude circle of length $L \approx 10\,000\,\mathrm{km}$, where $\beta \approx 2 \times 10^{-11}\,\mathrm{m^{-1}\,s^{-1}}$. For what range of values of U, a uniform westerly zonal flow, might you expect to see a stationary atmospheric response excited by this topography?

DPA.4

Outline the main assumptions behind Held & Hou's simple model of a nearly inviscid, axisymmetric Hadley circulation.

Consider this model on an equatorial β-plane, so that $f = \beta y$. For a radiative equilibrium temperature profile with latitude given by

$$\Theta_E = \Theta_{E0} - \frac{\Delta\theta}{a^2}y^2 ,$$

where Θ_{E0} is a constant and a is the planetary radius, define a suitable form for m, the angular momentum per unit mass, and verify that conservation of absolute angular momentum in (geostrophic) thermal wind balance leads to

$$\frac{\partial\theta}{\partial y} = -\frac{2\Omega^2\theta_0}{a^2 gH}y^3 ,$$

where Ω is the planetary rotation rate, g the acceleration due to gravity, and H and θ_0 are constants. Hence obtain an expression for the horizontal profile of temperature Θ_M consistent with angular momentum conservation.

By assuming that the Hadley circulation extends between $y = \pm Y$ in a channel with boundaries at $y = \pm L$, where $L > Y$ and $\Theta = \Theta_E$ for $|y| \geq Y$, and that there is no net heating/cooling of air parcels, show that Y is given by

$$Y = L(5R/3)^{1/2}$$

where

$$R = \frac{g\Delta\theta H}{\Omega^2 L^2 \theta_0} .$$

Using the approximate values in the following table for equator–pole temperature difference ΔT and a representative atmospheric temperature T in order to estimate $\Delta\theta/\theta_0$, calculate Y with the other values given for each planet. Determine the predicted extent of the Hadley circulation in latitude using the planetary radii a. How do they compare with the observed circulations? Note that the atmospheric scale height has been used for H, the actual Hadley circulation may be deeper in some cases.

	ΔT [K]	T [K]	a [10^6 m]	Ω [10^{-5} s^{-1}]	g [m s^{-2}]	H [10^3 m]
Venus	20	240	6.051	0.030	8.60	5
Titan	10	90	2.575	0.456	1.35	18
Mars	40	225	3.397	7.09	3.72	11
Earth	40	255	6.378	7.29	9.81	8
Jupiter	20	125	71.40	17.6	22.9	20