Physics of Atmospheres and Oceans: Class Question Sheets

GEOPHYSICAL FLUID DYNAMICS 1

G1.1. If a ball is thrown a horizontal distance of 100m at 30° latitude in 4 seconds, what is its sideways deflection due to the Coriolis force?

G1.2. Perform a scale analysis for the horizontal and vertical momentum equations for oceanic motion of Atlantic ocean scale:

$$U \sim 10 \text{ cm s}^{-1},\, W \sim 1 \text{ mm s}^{-1},\, H \sim 4 \text{ km},\, T \sim 1 \text{ month}.$$

(Choose an appropriate horizontal length scale, and justify your choice; neglect friction. Note that the sizes of horizontal and vertical pressure gradients should be inferred from the other terms.) Estimate the Rossby number for this flow. What U or L scales would be required for the flow to be significantly non-geostrophic?

G1.3. Estimate the Rossby numbers for the following flows, and comment:

(a) A hurricane at 20°N, with windspeed $\sim 70\,\text{m}\,\text{s}^{-1}$ and horizontal lengthscale $\sim$ 100 km.
(b) A tornado in the American Midwest, with windspeed $\sim 100\,\text{m}\,\text{s}^{-1}$ and horizontal lengthscale $\sim$ 100 m.
(c) Flow in a bathtub vortex.
(d) A "Polar Low" at 60°N, with windspeed $\sim 15\,\text{m}\,\text{s}^{-1}$ and horizontal lengthscale $\sim$ 500 km.

G1.4. Consider a vortex in which the flow is steady, horizontal and independent of height, with circular streamlines, in a Cartesian frame rotating with angular velocity Ω about the vertical. Neglecting friction, show that

$$\frac{u^2}{r} + 2\Omega u = \frac{1}{\rho}\frac{dp}{dr}$$

where r is the radial distance from the centre of the vortex, $u(r)$ is the (tangential) wind speed, $p(r)$ is the pressure and ρ is the density (assumed constant).

Hence explain why highs are regions of weak pressure gradients and gentle winds, but lows can have large pressure gradients and strong winds. Is the geostrophic wind an overestimate or underestimate of the wind in a low pressure system?

G1.5. Atmospheric fronts are narrow regions of large horizontal temperature gradient. A simple model takes the front to be a sloping surface across which the temperature and along-front wind are discontinuous (with warm air overlying cold), but the pressure and cross-front wind are continuous. Take the y-axis along the front, the x-axis pointing towards the cold air, and apply the hydrostatic and geostrophic wind relationships to the region AB in Figure 1:

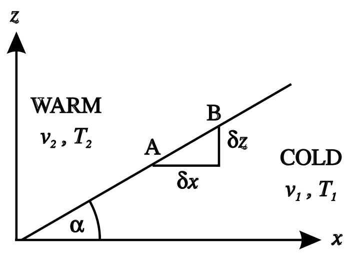


Figure 1.

Hence show that the slope α at any level is related to the temperatures T_1, T_2 and along-front winds v_1, v_2 at that level by

$$(T_2 - T_1) g \tan\alpha = f(v_1 T_2 - v_2 T_1).$$

If $T_2 - T_1 = 3\text{K}$ and $v_1 - v_2 = 10$ m s^{-1}, estimate α at 50°N latitude.

G1.6. Starting with the geostrophic and hydrostatic equations in pressure coordinates, derive the thermal wind relationship in the form

$$\frac{\partial}{\partial p}(u, v) = \frac{R}{fp}\left(\frac{\partial T}{\partial y}, -\frac{\partial T}{\partial x}\right),$$

making clear which variables are held constant in each partial derivative. (R is the gas constant for dry air.) How does this equation differ from the $z-$coordinate version, where z is geometric height?

Introduce the variable $Z = \ln(p_0/p)$, where p_0 is a constant reference pressure, and derive the thermal wind relationship using Z as a vertical coordinate.

The temperatures in the following table are derived from satellite measurements for latitudes 50°N and 40°N and longitude 0°E. Given that the eastward wind at (45°N, 0°E) and a pressure of 10 mb is $25\,\text{m}\,\text{s}^{-1}$, estimate the eastward wind at (45°N, 0°E) at a pressure of 1 mb.

Pressure/mb	10	1
(Temperature at 50°N)/K	217	252
(Temperature at 40°N)/K	224	261

[Use the trapezoidal rule $\int_a^b F(Z)\,dZ \approx \frac{1}{2}\big[F(a) + F(b)\big] \times (b - a)$.]

G1.7. Figure 2 is a typical "weather map" of a developing cyclone (in the northern hemisphere) showing contours (isobars) of surface pressure in millibars and the regions of cold and warm air at the surface (the latter is called the 'warm sector'). Giving physical justification, mark the directions of the geostrophic winds by arrows, and the positions of the cold (▲▲▲) and warm (◓◓◓) fronts. (Note: warm air follows cold in a warm front, and vice versa in a cold front.) Check that the changes in wind direction at the fronts agree with the results of question G1.5.

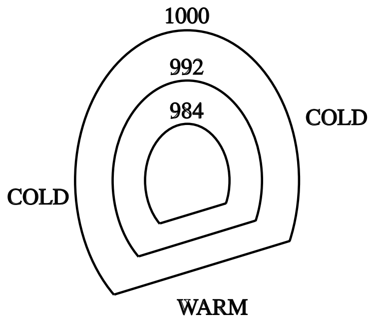


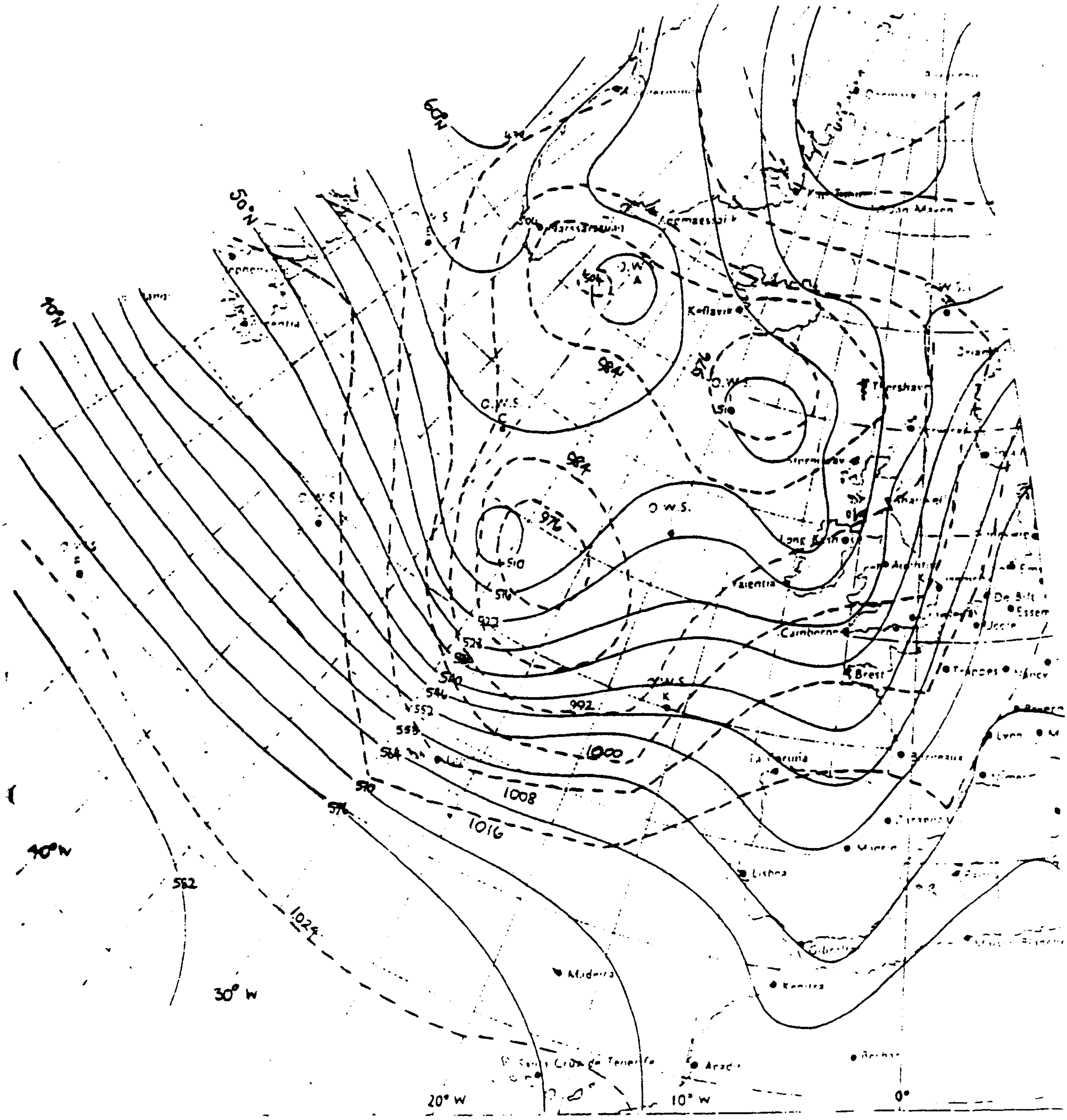
Figure 2.

G1.8. Figure 3 is a North Atlantic chart, showing surface pressure (dashed contours, in millibars) and the height of the 500 mb surface (solid contours, in dekametres; 1 dekametre = 10 m).

Show that near 1000 mb the pressure drops by about 8 mb for every 60 m of vertical ascent (take the density of air at the surface $\simeq 1.3$ kg m^{-3}). Hence show that the surface isobars can be roughly reinterpreted as isopleths of the 1000 mb height, and re-label them accordingly (in dekametres). For the sector 10°W – 35°W, 35°N – 55°N, draw in contours of the thickness of the 1000 mb – 500 mb layer, and shade the region where the mean temperature of this layer is largest. What is the name given to this region, and what is its mean temperature?

Shade in the region of strongest geostrophic winds at 500 mb, and estimate the maximum geostrophic wind speed there.

Figure 3. North Atlantic Chart for 00 hrs on 31 January 1974.



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