Physics of Atmospheres and Oceans - Class Question Sheets

Michaelmas Term: INTRODUCTORY PROBLEMS

Those intending to do this Major Option should complete the following problems during the vacation, for handing in at the beginning of Michaelmas Term. (These problems are mostly based on the B3 Atmospheric Physics course.)

Recommended further reading: *An Introduction to Atmospheric Physics*, D G Andrews (CUP), *Physics of Atmospheres*, J T Houghton (CUP).

1. In a thought-experiment the temperature of the atmosphere is decreased by 1 K everywhere and the heat thereby released is given to the top 100 m of the ocean, whose temperature rises by an amount δT everywhere. Find δT. Comment on the possible implications of this result for climate change.

[Take the specific heat capacity of sea water to be 4.2 and c_p of air to be 1.0, each in units of $\mathrm{kJ\,K^{-1}\,kg^{-1}}$. Take the radius of the Earth to be 6400 km.]

2. A balloon is required to carry an instrument payload to a height of 30 km at a latitude of 40° N in June, corresponding to a typical atmospheric pressure of about 1225 Pa and temperature of about 233 K. The total mass of the payload plus the balloon is 150 kg. What approximate radius of balloon is needed, assuming that after being charged with a measured "bubble" of helium at launch it just becomes spherical at float altitude?

What are the relative merits and disadvantages of balloons, satellites and aircraft as instrument platforms?

3. Show that, if there is a uniform lapse rate $-dT/dz = \Gamma$, the pressure in the atmosphere is given by

$$p(z) = p_0 \left(1 - \frac{\Gamma z}{T_0}\right)^{Mg/\Gamma R}$$

where M is the molar mass of air and R is the molar gas constant. Calculate the height at which the pressure is 0.1 of its surface value (p_0) assuming a surface temperature (T_0) of 290 K and (i) a uniform lapse rate of $10\,\mathrm{K\,km^{-1}}$ and (ii) a uniform temperature of 290 K.

4. What is meant by the *stability* of a 'parcel' of air, with respect to its vertical motion? Investigate the criterion for stability as follows.

By considering a rising parcel of air, give a physical definition of the *dry adiabatic lapse rate* Γ_d. Write down the expression for Γ_d in terms of familiar physical quantities.

Consider a parcel at height z and temperature T that is originally in equilibrium with the surrounding air. It is displaced adiabatically through a vertical distance δz : what is its new temperature? Assuming the surrounding air has lapse rate Γ, what is the temperature of the surrounding air at the same height as the displaced parcel? (Ignore any disturbances to the surrounding air due to the parcel's displacement.)

Assuming that the parcel has the same pressure as its surroundings and using the ideal gas law, find the difference in density between the parcel and its surroundings. Hence show that the parcel is stable if $\Gamma < \Gamma_d$.

5. Show that the vertical acceleration of a parcel of air of density $\rho_{\rm p}$ in an atmosphere of density ρ is given by

$$\left(\frac{\rho}{\rho_{\rm p}} - 1\right) g \ .$$

Given that the parcel moves adiabatically, show that for a small vertical displacement δz from equilibrium

$$\frac{d^2\,\delta z}{dt^2} + N^2\,\delta z = 0 \ , \qquad \text{where} \qquad N^2 = \frac{g}{T}\left(\frac{dT}{dz} + \frac{g}{c_p}\right)$$

and $-dT/dz$ is the lapse rate of the surrounding atmosphere.

Discuss solutions of this equation when $N^2 < 0$, $= 0$ and > 0, and explain how they relate to question 4. For $N^2 > 0$, calculate the period of the oscillation given a lapse rate of 6.5 K km^{-1} and $T = 270$ K. [This oscillation is a particular case of an *internal gravity wave* and its angular frequency is called the buoyancy frequency, or Brunt-Väisälä frequency.]

6. Calculate the times taken for water drops of radii 1 μm, 10μm and 100μm to fall a distance of 500 m in air at terminal velocity.

[Assume Stokes's Law, that the viscous force on a spherical drop of radius a and speed v is $6\pi\mu a v$. Take the dynamic viscosity μ for air to be 1.7×10^{-5} kg m^{-1}s^{-1} and the density of air at the relevant altitude to be 1.0 kg m^{-3}.]

Check the validity of your calculation by estimating the Reynolds Number for each radius.

7. Write down an expression, in terms of a vertical integral, for the *optical depth* $\tau(z)$ corresponding to a specific absorbing gas and a specific wavelength. Give the corresponding expression for the *transmittance* $\mathcal{T}(z)$ from height z to space.

Solve the monochromatic radiative transfer equation in the form

$$R_\infty = \int_0^\infty B \frac{d\mathcal{T}}{dz}\, dz + B(0)\mathcal{T}(0) \ ,$$

where R_∞ is the radiance leaving the top of the atmosphere and $B(z)$ is the Planck black-body function corresponding to the temperature at height z. (Assume that the ground emits as a black body.)

Give a physical interpretation of each of the terms in this equation.

8. Find expressions for the optical depth $\tau(z)$ at height z, and the transmittance $\mathcal{T}(z)$ to space from height z, for an absorbing gas whose density varies as

$$\rho_{\rm a}(z) = \rho_{\rm a0} \exp(-z/H_{\rm a}) \ ,$$

where $H_{\rm a}$ is a constant scale height, at a wavelength where the absorption coefficient is k. Find an expression for the 'weighting function' $d\mathcal{T}/dz$, and show that this is a maximum at the height where $\tau = 1$.

Sketch the optical depth, the transmittance and the weighting function as functions of height.

Optional: Comment on the significance of the weighting function for remote sounding of the atmospheric temperature profile.

[Updated 9 June 2008]