**Physics of Atmospheres and Oceans: Class Question Sheets**

OCEANS AND CLIMATE

OC1. The typical magnitude of the wind stress associated with the easterly trade winds blowing over the tropical Pacific is $0.1\,\mathrm{N\,m^{-2}}$.

Deduce the direction and magnitude of the Ekman transports at 4°S and 4°N. Hence estimate the net volume of water upwelling between 4°S and 4°N, expressing your answer in Sverdrups. What is the mean rate of upwelling in meters per year? You can assume that the width of the tropical Pacific is approximately 15 000 km.

During an El Niño year, the trade winds weaken markedly. What will be the impact on the sea surface temperatures in the eastern equatorial Pacific and why?

OC2. A rectangular basin, extending from $x = 0,\ L$ and $y = 0,\ L$, is forced by a zonal wind stress which varies with latitude according to:

$$\tau_s^{(x)} = -\tau_0 \cos\left(\frac{2\pi y}{L}\right).$$

Sketch the circulation predicted by Sverdrup balance, clearly stating any boundary conditions that you assume.

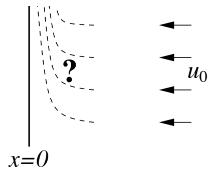
Show that the transport of each gyre is roughly

$$T \approx \frac{2\pi\tau_0}{\rho_0\beta}.$$

Estimate $T$ assuming a maximum wind stress of $0.1\,\mathrm{N\,m^{-2}}$ and typical values for the remaining parameters. Express your answer in Sverdrups.



OC3. Consider a uniform inflow, $u = -u_0$ $(x \to \infty)$, impinging on a boundary at $x = 0$.




Assume that the flow satisfies the steady-state vorticity equation for a homogeneous ocean,

$$\mathbf{u}\cdot\nabla\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} + \beta y\right) = -r\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right),$$

in which wind forcing has been neglected.

Write down the approximate form of this vorticity equation in the limit in which inertia can be neglected. Derive the solution for the streamfunction in this frictional limit,

$$\psi \approx u_0 y\left(1 - e^{-x/\delta_s}\right),$$

where $\delta_s = r/\beta$ is the width of a frictional (Stommel) boundary current. Calculate $\delta_s$ assuming a frictional spin-down time-scale of 1 year and a typical value for $\beta$.

Now consider the alternative limit in which friction can be neglected. Show that

$$\nabla^2\psi + \beta y = q(\psi)$$

and deduce the functional form of $q(\psi)$ from the boundary condition at $x \to \infty$. Hence derive the solution for the streamfunction in this inertial limit,

$$\psi \approx u_0 y\left(1 - e^{-x/\delta_i}\right),$$

where $\delta_i = \sqrt{u_0/\beta}$ is the width of an inertial boundary current. Assuming a value of $u_0$ consistent with the westward flow in a Sverdrup gyre, estimate $\delta_i$.

Using your answers, deduce which process, friction or inertia, is the more likely to set the width of western boundary currents in the ocean.

OC4. Evaporation exceeds freshwater input to the Mediterranean (i.e., input from rivers and precipitation) by an amount $E = 7\times10^4\,\mathrm{m^3s^{-1}}$. You can assume that water flows into the Mediterranean from the Atlantic at a rate $q_{in}$ and with a mean salinity $S_{in} = 36.3\,{}^o/_{oo}$, and that water flows out of the Mediterranean at a rate $q_{out}$ and with a mean salinity $S_{out} = 37.8\,{}^o/_{oo}$.

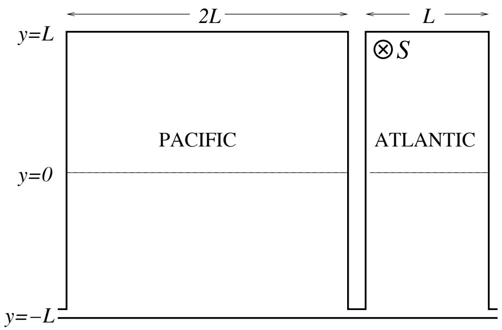
By writing down volume and salt budgets for the Mediterranean, show that

$$q_{out} = \frac{E\,S_{in}}{S_{out} - S_{in}}.$$

Estimate how much water is flowing into the Atlantic from the Mediterranean.

The Mediterranean contains about $3.8\times10^6\,\mathrm{km^3}$ of water. Estimate how long it takes to recycle this entire body of water.

OC5. Deep convection results in a localised source, $S = 15\,\mathrm{Sv}$, of abyssal water in the northwestern corner of an idealised "Atlantic" basin, as sketched below. The "Atlantic" basin, of dimensions $L\times 2L$, is connected to an idealised "Pacific" basin, of dimensions $2L \times 2L$, as sketched. You can assume that the abyssal ocean is a slab of uniform thickness, the Coriolis parameter is given by $f = \beta y$, where the equator, $y = 0$, is located at the centre of the basins, and $L = 5000\,\mathrm{km}$.






Assuming $S$ is balanced by uniform upwelling, $w^*$, over the remainder of the basins, write down a formula for $w^*$. Over what distance does the abyssal fluid upwell in 1 year?

How much water upwells in the "Atlantic" and how much upwells in the "Pacific"?

Write down the equations of geostrophic balance. By substituting these into the continuity equation, derive the expression for the depth-integrated meridional velocity over the abyssal layer, valid within the interior of the basin:

$$\int v\,dz = w^* y.$$

Explain why boundary currents are required to close the circulation. On which side of the basin would you expect to find the boundary currents and why?

By considering the net northward transport required at each latitude, and the northward transport in the basin interior, show that the northward transport of the boundary current is

$$T_{BC} = -\frac{S}{6}\left(5 + \frac{2y}{L}\right)$$

in the Atlantic and

$$T_{BC} = \frac{S}{3}\left(1 - \frac{2y}{L}\right)$$

in the Pacific.

What is the maximum boundary current transport (in either basin)?

Why does the direction of the boundary current reverse at $y = L/2$ in the Pacific?

Sketch the horizontal circulation patterns predicted by this model.

How would the circulation differ in this model if all of the upwelling were to occur in the circumpolar belt of the "Southern Ocean"?

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