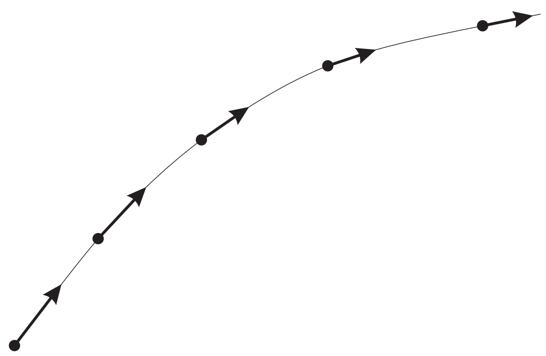
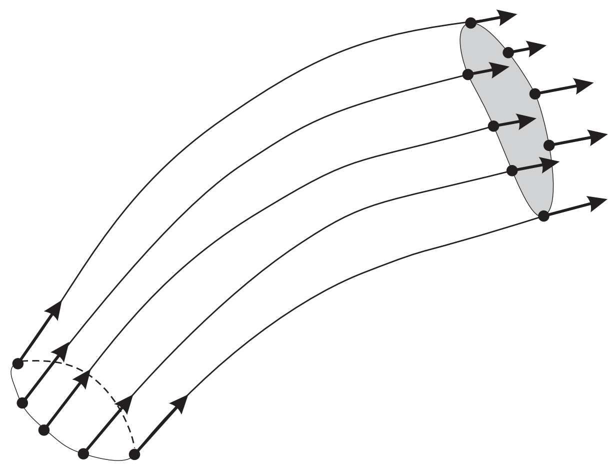
Vortex lines and vortex tubes [Not examinable!]

At any instant t a *vortex line* is defined as a line that, at each position $\boldsymbol{r}$, points in the direction of the vorticity vector $\boldsymbol{\omega}(\boldsymbol{r}, t)$ at that position:



A *vortex tube* is formed from a bundle of vortex lines (most arrows omitted here, for clarity):



The flux of vorticity through the tube (the 'strength' of the tube) is

$$\Gamma_{\rm tube} = \int_{\rm S} \boldsymbol{\omega} \cdot d\boldsymbol{S}$$

for any cross-section S of the tube.

NB: $\Gamma_{\rm tube}$ is the same for *all* cross-sections of the tube. [Prove this by drawing 2 such cross-sections, applying the divergence theorem (Gauss' theorem) to the volume between them and the tube walls, and using $\nabla \cdot \boldsymbol{\omega} = 0$.]

Note that $\Gamma_{\rm tube} = \Gamma_{\rm C}$, where C is the perimeter of S.

How do vortex tubes move in time?

It can be shown (e.g. Acheson pp. 162-4) that if $\nu = 0$, the vortex tube *moves with the fluid flow*.

By Kelvin's CT, the strength of the vortex tube remains constant in time.

By letting the cross-sectional area $\to 0$, we see that vortex lines also move with the fluid when $\nu = 0$.