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LETTERS

Saturn's rotation period from its atmospheric planetary-wave configuration

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The rotation period of a gas giant's magnetic field (called the System III reference frame) is commonly used to infer its bulk rotation[1]. Saturn's dipole magnetic field is not tilted relative to its rotation axis (unlike Jupiter, Uranus and Neptune), so the surrogate measure of its long-wavelength (kilometric) radiation is currently used to fix the System III rotation period[2]. The period as measured now by the Cassini spacecraft is up to ~7 min longer[3] than the value of 10 h 39 min 24 s measured 28 years ago by Voyager[2]. Here we report a determination of Saturn's rotation period based on an analysis of potential vorticity. The resulting reference frame (which we call System IIIw) rotates with a period of 10 h 34 min 13 ± 20 s. This shifted reference frame is consistent with a pattern of alternating jets on Saturn that is more symmetrical between eastward and westward flow. This suggests that Saturn's winds are much more like those of Jupiter than hitherto believed[4].

Saturn's bulk rotation period is not expected to have changed appreciably over the course of the last few decades (the interior is nearly isentropic and there is no plausible way to alter the moment of inertia). A good surrogate of the rotation period should therefore likewise remain constant. The case for this role to be played by periodicities in Saturn's kilometric radiation (SKR) has been considerably weakened now that the Cassini spacecraft has found a value[3,5] of 10 h 47 min 6 s (here designated 'System IIIa'; Table 1), which is markedly different to that determined by Voyager. Moreover, this period appears to be changing[3] even during the short time interval that Cassini has been in orbit around Saturn. It seems clear, therefore, that the SKR periodicities are affected by factors that partially decouple the external magnetic field from the deep interior and allow it to 'slip', possibly in association with a centrifugally driven instability in Saturn's plasma disk[3,6].

Table 1 | Saturn's rotation periods relative to Voyager System III

Name of reference frame	Rotation period (h min s)	Frame zonal velocity at the equator in System III ($m\,s^{-1}$)
System III[1]	10 39 24 ± 7	0
System IIIa[3]	10 47 06 ± 40	−123 ± 11
System IIIb[7]	10 32 35 ± 13	112 ± 4
System IIIw	10 34 13 ± 20	85 ± 6

To determine System IIIw, a cross-section of zonally averaged quasi-geostrophic potential vorticity in latitude and pressure (height) was derived from the Cassini geostrophic thermal wind and temperature measurements, as described fully elsewhere[16,17]. For vertical levels located in the upper troposphere, bands of latitude up to 10° wide and consistent with marginal stability according to Arnol'd's second stability theorem were identified from the condition that $d\bar{q}/d\bar{\Psi} < 0$, where $\bar{\Psi}$ is the geostrophic streamfunction for the mean zonal flow (obtained from a latitudinal integral of the geostrophic zonal wind). Within each marginally stable latitude band so identified, a linear regression line between $\bar{u}$ and $\bar{q}_y$ of the form $\bar{u} = L_D^2 \bar{q}_y + \alpha$ was obtained by a least squares fit. This enabled a profile of $\alpha(\phi)$ to be obtained, evaluated at the centre of each latitude band for each pressure level, which could then be converted to an angular velocity profile $\omega(\phi)$ as described in the text, taking full account of the oblate spheroidal shape of the planet. An average value for the angular velocity of System IIIw was obtained from a weighted mean of $\omega(\alpha_i)$ over the range of pressures 150–300 hPa and all relevant latitudes, using weights $1/s_i^2$, where s_i is the standard regression error of $\omega(\alpha_i)$. The quoted uncertainty on the rotation period is an estimate of the 1σ weighted standard error on the mean.

In a recent study[7], a different approach was used to estimate Saturn's rotation period by combining measurements of its gravity field with radio occultation and zonal wind measurements from the Pioneer and Voyager missions. This resulted in a value of 10 h 32 min 35 ± 13 s (hereafter referred to as 'System IIIb'; Table 1), which is faster than any of the earlier estimates, yields peak winds on the equator that are more than 100 $m\,s^{-1}$ slower than previously considered, and yields alternating jets in the eastward and westward directions of more or less equal magnitude, much more like the pattern seen on Jupiter. However, this estimate is not yet widely accepted as the 'true' interior rotation period pending further observational or theoretical evidence.

Here we introduce an independent approach to the estimation of Jupiter's and Saturn's bulk rotation periods, based on considerations of their dynamic meteorology. From analyses of Voyager cloud-tracked winds on Jupiter, the distribution of zonally averaged potential vorticity, $\bar{q}$, appears to approach marginal stability[8–10] with respect to a nonlinear stability theorem due to Arnol'd[11,12]; in particular, Arnol'd's second stability theorem implies that perturbations will be stable if

$$\left(\frac{\bar{u}-\alpha}{\partial\bar{q}/\partial y}\right) \geq L_D^2 \qquad (1)$$

where $\bar{u}$ is the zonal-mean zonal velocity, y is the northward coordinate, α is a constant that corresponds to a shift in the reference frame, and L_D is an intrinsic length scale that may vary with latitude (the first baroclinic radius of deformation). Rossby waves travel westward or eastward relative to the flow depending on whether $\partial\bar{q}/\partial y \equiv \bar{q}_y$ is positive or negative, respectively; on Earth, $\bar{q}_y$ is dominated by the planetary vorticity gradient, known as β, which is positive in both hemispheres, thus Rossby waves generally travel westward relative to the flow. However, on Jupiter and Saturn, the curvature of their jet streams is much stronger than on Earth, and $\bar{q}_y$ is negative at many latitudes. The longest Rossby waves, which are among the most deeply rooted of atmospheric disturbances[13], are also the fastest, and in the marginally stable case they propagate upstream with speed $-L_D^2\bar{q}_y$ relative to the flow such that they just cancel their advection by the wind[9]. Thus, α represents the speed of these waves in the chosen reference frame.

The tendency towards marginal stability in the sense of the limiting value of equation (1) has recently been demonstrated in three-dimensional numerical models of gas-giant atmospheres[14] (the abyssal circulation underlying a gas giant's spherical-shell atmosphere, or 'weather layer', may not be governed by equation (1) because of the effects of convection and full spherical geometry in the interior[15]). On the observational front, much improved zonal-mean maps of

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potential vorticity (quasi-geostrophic and Ertel) have recently been derived[16,17] in the upper tropospheres and lower stratospheres of both Jupiter and Saturn, making use of both cloud-tracked velocities and retrievals of thermal structure and composition from the Voyager and Cassini missions. These results confirm that substantial bands of latitude on both planets are broadly consistent with the marginally stable form of equation (1) (with an equality). Moreover, equation (1) was used to deduce the variation of L_D with latitude from the gradient of the linear regression between $\bar{u}$ and $\bar{q}_y$. The primary advance here is the recognition that this same linear regression can independently solve for α itself, and hence the unique reference frame in which the longest Rossby waves are stationary.

Figure 1 compares the latitudinal variation of α on Jupiter and Saturn, derived from the quasi-geostrophic potential vorticity profiles obtained from Cassini data[16,17], relative to their traditional System III reference frames, converted to the respective angular velocity (that is, $\omega(\alpha) = \alpha/(a(\phi)\cos\phi)$, where $a(\phi)$ is the radius of the planet as measured from its centre, allowing for its ellipticity, at planetocentric latitude ϕ). The quasi-geostrophic theory underlying equation (1) is not valid near the equator; taking a variance-weighted average of $\omega(\alpha)$ across all latitudes $>15°$ from the equator, excluding the equatorial jet, and over pressure levels from 150 to 300 hPa, leads to a reference frame on Jupiter (hereafter denoted 'System IIIw') that differs in rotation period from Voyager's System III by just 18 ± 1 s, which is statistically consistent with only a very small shift (about $-6.6\ \mathrm{m\,s^{-1}}$ at the equator). For Saturn, if we restrict the coverage to latitudes $>20°$ from the equator, also excluding the equatorial jet, the mean difference in period between System III and System IIIw is 311 ± 20 s. Although the formal uncertainty is higher than for the same analysis applied to Jupiter, this does suggest that dynamical stability considerations on Saturn favour a reference frame that rotates over 5 min faster than Voyager System III, which corresponds to over $80\ \mathrm{m\,s^{-1}}$ faster at the equator (Table 1). When viewed in this frame, Saturn's pattern of alternating zonal jets adopts a more symmetrical appearance (Fig. 2a), with more or less equal amplitude eastward and westward jets at middle and high latitudes. Such a symmetric jet pattern resembles the pattern on Jupiter much more closely than when viewed in Saturn's traditional System III frame (compare Fig. 2b).

The notion that an atmospheric flow might adjust towards a state of marginal dynamical stability is not new. Such a scenario has been suggested, for example, as an explanation for the static stability structure of the Earth's atmosphere through the process of 'baroclinic adjustment'[18]. In the case of Jupiter and Saturn, barotropic processes and instabilities are likely to be at least as important as baroclinic ones. This would lead to a dynamical equilibration between upscale cascade processes, forcing zonal jets towards an ever stronger, dynamically unstable structure, and the action of large-scale instabilities in restoring the flow towards a marginally stable structure[9,19,20]. A physical interpretation[8,9,21] of α associates it with the speed of the longest-wavelength Rossby wave for almost barotropic waves. This is also consistent with the theoretical finding[21] that marginally unstable Jovian barotropic Rossby waves are approximately stationary. Such waves may 'sense' planetary-scale disturbances deep in the interior[9], although we do not yet know the precise nature of such a teleconnection.

On the basis of the atmospheric planetary-wave configuration (potential vorticity and zonal wind) observed on Jupiter, we have deduced that its longest Rossby waves are frequently coherent across its adjacent alternating jets and almost stationary relative to its bulk planetary rotation period. To the extent that this System IIIw method is thus verified, we have been able to mitigate Saturn's lower signal-to-noise ratio (for both its magnetic field and atmosphere) by applying the same approach. The main result is that the zonal-wind profiles for Saturn and Jupiter now appear to be qualitatively similar. Accurate knowledge of Saturn's rotation rate is necessary not only to understand the dynamics of its atmosphere, but it is also required to infer the planet's interior structure and dynamical state, deduce whether it has a heavy element core, and constrain theories of its origin and evolution. The new rotation period obtained here, for example, would indicate[22] a mean density for the planet of $\rho_0 = 686.895\ \mathrm{kg\,m^{-3}}$ and a value for its J_2 gravitational moment of

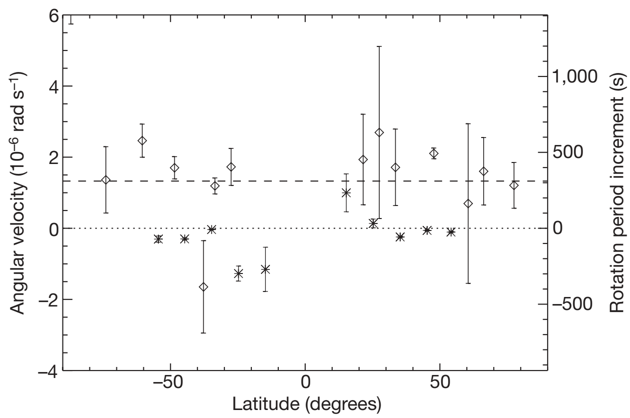

Figure 1 | Variation of limiting planetary wave angular velocity with latitude for Saturn and Jupiter. Shown are latitudinal profiles of the 'long wave' angular velocity, $\omega(\alpha) = \alpha/(a(\phi)\cos\phi)$ (where $a(\phi)$ is the planetary radius and ϕ is planetocentric latitude), and corresponding increment in rotation period for Saturn (open diamonds) and Jupiter (stars) from the corresponding System III at pressure levels of 268 hPa (Jupiter) and 236 hPa (Saturn). Error bars on the points are representative of the formal (1σ) uncertainties in the linear regression coefficient, and indicate some significant variation in the degree to which the relationship between $\bar{u}$ and $\bar{q}_y$ is well represented by the linear form $\bar{u} \approx L_D^2 \bar{q}_y + \alpha$. Where the error bars are small, this assumed relationship is a relatively good fit, lending some confidence to the hypothesis that winds in this latitude band are close to marginal stability. At some other latitudes, however, the fit is much poorer, indicating that an assumption of marginal stability may not be valid[16,17]. The latitudes where the fit is relatively poor also seem[16,17] to correspond with dynamically active regions, with an increased presence of large-scale wave activity or the formation of oval vortices and other disturbances. The dashed line represents the angular velocity of System IIIw for comparison.

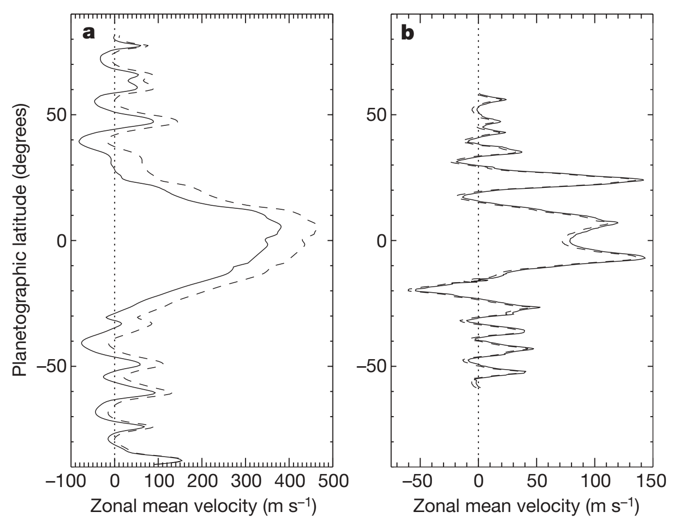

Figure 2 | Comparison of cloud-level zonal velocity profiles on Saturn and Jupiter as viewed in System IIIw and System III. **a**, **b**, Variations of cloud-level zonal velocity on Saturn (**a**) and Jupiter (**b**) with latitude, as viewed in the corresponding System IIIw reference frame (black solid line), and as viewed in the conventional Voyager System III frame (dashed line). Cloud-level velocities for Saturn were obtained from a combination of Voyager and Cassini data[17] whereas those for Jupiter are from the Cassini Imaging Sub-System (ISS)[23].

0.0162919, both of which may be amenable to accurate observational verification by future space missions.

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