

Inverse Methods for Atmospheric Sounding: Theory and Practice

Clive D. Rodgers

September 21, 2010

ERRATA for the 2008 reprinting

Page 31, Equation 2.58:

The term $\mathbf{G}\mathbf{y}$ should clearly be $\mathbf{G}(\mathbf{y} - \mathbf{K}\mathbf{x}_a)$, from Eq. (2.44). However it seems neater to rewrite the sentence and equation as:

For the moment simply note, from Eq. (2.44), that it relates the expected state $\hat{\mathbf{x}}$ to the true state \mathbf{x} :

$$\hat{\mathbf{x}} - \mathbf{x}_a = \mathbf{A}(\mathbf{x} - \mathbf{x}_a) + \mathbf{G}\boldsymbol{\epsilon} \quad (1)$$

[Thanks to Jörn Ungermann]

Page 74, Equation 4.53

This equation should read:

$$\mathbf{G} = \mathbf{S}_a \mathbf{K}^T (\mathbf{K} \mathbf{S}_a \mathbf{K}^T + \gamma \mathbf{S}_\epsilon)^{-1} \quad (2)$$

[Thanks to Mick Christi]

Page 97, Equation 5.47

The the term \mathbf{x}_a in the fourth line of the equation should be replaced by \mathbf{x}_{j-1}^{i+1} giving:

$$\mathbf{x}_j^{i+1} := \mathbf{x}_{j-1}^{i+1} + \mathbf{S}_{j-1} \mathbf{k}_j [y_j - F_j(\mathbf{x}_i) - \mathbf{k}_i^T (\mathbf{x}_i - \mathbf{x}_{j-1}^{i+1})] / (\mathbf{k}_j^T \mathbf{S}_{j-1} \mathbf{k}_j + \sigma_j^2) \quad (3)$$

[Thanks to Jörn Ungermann]

Page 108, Section 6.3

This section is a mess. As well as about seven typos, it is not clearly written. Here is another attempt for the part of this section on page 108:

However it is more convenient to use the alternative formulation in terms of singular vectors, mainly because singular vectors are orthogonal. Put

$$\mathbf{C}^{-1} \simeq \mathbf{U}'_t \mathbf{\Lambda}_t^{-2} \mathbf{V}'_t{}^T, \quad (4)$$

where \mathbf{U}'_t and \mathbf{V}'_t are matrices of left and right singular vectors of \mathbf{C} truncated to t columns, and the truncated matrix of singular values has been written as $\mathbf{\Lambda}_t^2$ for reasons which will be apparent later. This gives

$$\hat{\mathbf{x}} = \mathbf{W} \mathbf{U}'_t \mathbf{\Lambda}_t^{-2} \mathbf{V}'_t{}^T \mathbf{y} \quad (5)$$

$$= \mathbf{W} \mathbf{U}'_t \mathbf{\Lambda}_t^{-2} \mathbf{V}'_t{}^T \mathbf{K} \mathbf{x} + \mathbf{W} \mathbf{U}'_t \mathbf{\Lambda}_t^{-2} \mathbf{V}'_t{}^T \boldsymbol{\epsilon}. \quad (6)$$

If $\boldsymbol{\epsilon}$ has covariance $\sigma_\epsilon^2 \mathbf{I}$ the error term will have covariance $\sigma_\epsilon^2 \mathbf{W} \mathbf{U}'_t \mathbf{\Lambda}_t^{-4} \mathbf{U}'_t{}^T \mathbf{W}^T$, each left singular vector contributing an independent error equal to a column of $\mathbf{W} \mathbf{U}'_t$ multiplied by a random normal variable with standard deviation $(\sigma_\epsilon / \lambda_i^2)$.

Rather than retaining a general representation function let us simplify matters by only considering $\mathbf{W} = \mathbf{K}^T$. In this case $\mathbf{C} = \mathbf{K} \mathbf{K}^T$ is symmetric and its singular vectors and eigenvectors are identical, and equal to \mathbf{U} , the left singular vectors of \mathbf{K} , $\mathbf{K} = \mathbf{U} \mathbf{\Lambda} \mathbf{V}^T$. Thus $\mathbf{U}'_t = \mathbf{V}'_t = \mathbf{U}_t$. The singular values (and eigenvalues) of \mathbf{C} are then the squares of the singular values of \mathbf{K} (hence the choice of $\mathbf{\Lambda}_t^2$ above). In this case

$$\hat{\mathbf{x}} = \mathbf{K}^T \mathbf{U}_t \mathbf{\Lambda}_t^{-2} \mathbf{U}_t^T \mathbf{y} \quad (7)$$

$$= \mathbf{V}_t \mathbf{\Lambda}_t^{-1} \mathbf{U}_t^T \mathbf{y} \quad (8)$$

$$= \sum_{i=1}^t \lambda_i^{-1} \mathbf{v}_i \mathbf{u}_i^T \mathbf{y} \quad (9)$$

because $\mathbf{K}^T \mathbf{U}_t = \mathbf{V}_t \mathbf{\Lambda}_t$. Note that Eq. (8) is equivalent to using a truncation of the pseudo inverse $\mathbf{K}^* = \mathbf{V} \mathbf{\Lambda}^{-1} \mathbf{U}^T$ based on $\mathbf{K} = \mathbf{U} \mathbf{\Lambda} \mathbf{V}^T$. The relation of the retrieval to the state vector is therefore

$$\hat{\mathbf{x}} = \mathbf{V}_t \mathbf{\Lambda}_t^{-1} \mathbf{U}_t^T \mathbf{K} \mathbf{x} + \mathbf{V}_t \mathbf{\Lambda}_t^{-1} \mathbf{U}_t^T \boldsymbol{\epsilon} \quad (10)$$

$$= \mathbf{V}_t \mathbf{V}_t^T \mathbf{x} + \mathbf{V}_t \mathbf{\Lambda}_t^{-1} \mathbf{U}_t^T \boldsymbol{\epsilon} \quad (11)$$

showing that the averaging kernel matrix is $\mathbf{A} = \mathbf{V}_t \mathbf{V}_t^T$. If $\boldsymbol{\epsilon}$ has covariance $\sigma_\epsilon^2 \mathbf{I}$ the error term will have covariance $\sigma_\epsilon^2 \mathbf{V}_t \mathbf{\Lambda}_t^{-2} \mathbf{V}_t^T$, the error patterns being given by $\mathbf{e}_i = (\sigma_\epsilon / \lambda_i) \mathbf{v}_i$.

[Thanks to Lars Hoffmann and Jörn Ungermann]

Page 144, Equation 9.12:

This equation should read:

$$L_n = (L_0 - \bar{B}_1) \tau_0 + \bar{B}_n + \sum_1^{n-1} \tau_i (\bar{B}_i - \bar{B}_{i+1}) \quad (12)$$

i.e change \bar{B}_{i-1} to \bar{B}_{i+1} .

[Thanks to Rachel Hodos]

Page 144, Equation 9.16:

This equation should read:

$$= B_i + \frac{B_{i-1} - B_i}{\chi_i} (1 - e^{-\chi_i}) - B_{i-1} e^{-\chi_i} \quad (13)$$

i.e. omit the $-B_i$ in the last term.

[Thanks to Scott Paine]

Minor clarifications and typographic errors

- Page 1, line 15: insert ‘is’ after ‘it’. (MC)
- Page 4, line 12: insert ‘of’ after ‘kind’. (MC)
- Page 9, line 2: replace ‘**b**’ by ‘**w**’. (MC)
- Page 10, line -9: replace ‘ $G_j(\zeta)$ ’ by ‘ $G_i(\zeta)$ ’. (MC)
- Page 13, line -10: separate ‘*vector*’ and ‘**x**’. (MC)
- Page 13, line -8: insert ‘than’ after ‘rather’. (MC)
- Page 17, line 13: replace ‘**k_j**’ by ‘**k_i**’ for consistency with page 16. (MC)
- Page 17, line 19: change to ‘called the *row space* of **K** or *range* of **K^T**’. (MC)
- Page 18, lines 13, 21 and 23: replace ‘**k_j**’ by ‘**k_i**’ for consistency with page 16. (MC)
- Page 21, line 1: delete ‘is used’. (MC)
- Page 22, line -16: insert ‘of’ after ‘*pdf*’. (MC)
- Page 23, lines 5 and 7: replace ‘ $P(y,x)$ ’ by ‘ $P(x,y)$ ’. (MC)
- Page 28, line 14: replace **T**^{-T} by **T**_ε^{-T}. (MC)
- Page 30, line -12: insert ‘be’ after ‘therefore’. (MC)
- Page 35, line -16: replace first ‘the’ by ‘a’. (MC)
- Page 39, Figure 2.5 caption: ‘Solid: eigenvalues of a non-diagonal model covariance in decreasing order of size; dotted: the same for a diagonal covariance matrix with the same total variance. (MC)
- Page 40, line 8: replace ‘on’ by ‘one’. (MC)
- Page 45, Equations (3.3), (3.5), and (3.7): replace **R**(...) by **R**[...]. (MC)
- Page 47, line 13: replace first ‘**a_i**’ by ‘**a_i^T**’. (MC)

- Page 48, line 4: insert ‘the’ before ‘bias’. (MC)
- Page 48, lines 8 and 9: replace ‘ \mathbf{b} ’ by ‘ $\hat{\mathbf{b}}$ ’. (MC)
- Page 50, line -5: insert ‘that’ after ‘terms’. (MC)
- Page 60, Figure 3.8 caption: replace ‘eight’ by ‘ten’. (MC)
- Page 61, line 3: replace $\mathbf{S}_\epsilon + \mathbf{K}_b^T \mathbf{S}_b \mathbf{K}_b$ by $\mathbf{S}_\epsilon + \mathbf{K}_b \mathbf{S}_b \mathbf{K}_b^T$ (MC)
- Page 62, Figure 3.11 caption: at end insert ‘Original sinusoid: dashed. Response: solid.’. (MC)
- Page 66, line 4: replace ‘ \mathbf{x}_0 ’ by ‘ \mathbf{x}_0 ’. (MC)
- Page 72, lines -6, -5: replace ‘maximum likelihood’ by ‘MAP’. (MC)
- Page 73, lines -14, -7: replace ‘measurement error’ by ‘retrieval noise’. (MC)
- Page 76, line -3: replace ‘(3.23)’ by ‘(4.59)’. (MC)
- Page 77, line 3: insert ‘the’ before ‘standard’. (MC)
- Page 77, line 8: delete the first ‘of’. (MC)
- Page 82, line 1: insert ‘treatment of’ after ‘exhaustive’. (MC)
- Page 84, Equation (5.4) : delete ‘2’ on LHS of Equation (5.4). (MC)
- Page 86, lines 2, -5: replace ‘moderately linear’ by ‘moderately non-linear’. (MC)
- Page 90, line 3: delete ‘so’ after ‘this’. (MC)
- Page 90, line -11: replace ‘then’ by ‘than’. (MC)
- Page 91, line 11: replace ‘11’ by ‘12’. (MC)
- Page 91, line 14: replace ‘moderately linear’ by ‘moderately non-linear’. (MC)
- Page 94, line -18: delete ‘be’ after ‘been’. (MC)
- Page 95, line -3: insert ‘it’ after ‘if’. (MC)
- Page 99, line -11/-10: replace ‘corresponding’ by ‘correspond’ . (MC)
- Page 103, Equation (6.8): insert minus on RHS of equation. (MC)
- Page 103, Equation (6.9): insert minus on LHS of equation. (MC)
- Page 105, Equation (6.12): change $i = \dots m$, to $i = 1, \dots m$,. (MC)
- Page 107, line 12: replace ‘Not’ by ‘Note’. (MC)

- Page 113, line -9: insert ‘a’ after ‘Thus’. (MC)
- Page 119, line 8: replace ‘give’ by ‘gives’. (MC)
- Page 120, line 11: replace ‘give’ by ‘gives’. (MC)
- Page 124, Equation (7.16): Remove the ‘hats’ from \mathbf{x}''_{t+1} and \mathbf{S}''_{t+1} in the first two equations. They are superfluous as the double prime already indicates a backwards estimate. Also, remove the commas in the subscripts on the left hand side of these equations. (MC)
- Page 145, Equation (9.19): insert minus before κ_k . (MC)
- Page 146, line 6: replace ‘is’ by ‘a’. (MC)
- Page 146, line 7: delete first ‘the’. (MC)
- Page 146, Equation (9.23): Encase expression on RHS after \sum in square brackets. (MC)
- Page 159–160: replace all occurrences of (a), (b) and (c) by (i), (ii) and (iii) respectively. (MC)
- Page 164, Subtitle 10.3.1.3: replace ‘a priori’ by ‘a posteriori’. (MC)
- Page 167, line 9: replace ‘measure’ by ‘measured’. (MC)
- Page 168, line 10: insert ‘this’ after ‘Unfortunately’. (MC)
- Page 168, line -11: insert a comma after ‘strictly’ (MC)
- Page 171, Equation (10.38): replace ‘+ ϵ_r + ϵ_s ’ by ‘+ $\mathbf{G}\epsilon_r$ + $\mathbf{G}\epsilon_s$ ’. (MC)
- Page 172, line -7: replace ‘moderately linear’ by ‘moderately non-linear’. (MC)
- Page 173, Equation (10.50): insert \mathbf{W}^T between terms in brackets (i.e. change ‘... \mathbf{W}] $^{-1}$ [$\hat{\mathbf{S}}^{-1}$...’ to ‘... \mathbf{W}] $^{-1}$ \mathbf{W}^T [$\hat{\mathbf{S}}^{-1}$...’). (MC)
- Page 186, line -2: delete second ‘priori’. (MC)
- Page 193, Equation (12.19): change $\mathbf{S}_{\epsilon_2}^T$ to \mathbf{S}_{ϵ_2} . (MC)
- Page 194, line 11: replace ‘our’ by ‘or’. (MC)
- Page 194, line 11: replace ‘compared’ by ‘compare’. (MC)
- Page 195, line 12: insert ‘in’ after ‘Substituting’. (MC)

(MC) My thanks to Mick Christi for his amazing diligence in finding all these!